Reliability-based optimization of evolutionary topology and automated generation of strut-and-tie models for 3D structures

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Abstract

This paper addresses reliability-based topology optimization (RBTO) coupled with the Smoothing-ESO (SESO) method for automated generation of optimal strut-and-tie models. The proposed approach handles with the generation of truss-like designs for three-dimensional problem, addressing the design of a single corbel and a deep beam with two openings. The reliability analysis is performed using the Reliability Index Approach (RIA) via First-Order Reliability Method (FORM) by inserting into the Topology Optimization (TO) approach the follow random variables geometry, volume, strength and compliance, considering the limit state functions maximum displacement and maximum von Mises stress imposed in the optimization procedure. The automatic generation of optimal 3D strut-and-tie models was obtained through the derivatives of the von Mises stress fields by finding the force paths where compression and tensil predominate, respectively, in the direction of the struts and ties for reinforcement insertion.

Keywords: Topology Optimization, Reliability, SESO, RBTO.

1 Introduction

The Strut-and-Tie models (STMs) is a powerful tool for designing reinforced concrete structures, especially when the stress flow is complex and nonlinear as known for the D-regions. The model is based on the assumption that the structure behaves like a truss, where the compressive and tensile forces are transferred through a system of struts-and-ties.

In the sense of TO approach, various techniques and algorithms have been proposed by researchers to develop STMs since the work proposed by [1]. Some of these techniques include the Evolutionary Structural Optimization (ESO) that was suggested by [2] and uses an evolutionary algorithm to optimize the size and shape of struts and ties in the STMs. The algorithm works by iteratively removing material from the design until a final configuration is reached that satisfies the structural requirements. Bidirectional Evolutionary Structural Optimization (BESO), proposed by [3] and [4] is an extension of the ESO method. It uses two optimization processes that work in opposite directions to achieve a balance between the structural performance and the weight of the design. Performance-based optimization (PBO) method, proposed by [5] and [6], uses an optimization approach to simultaneously optimize both the structural performance and the cost of the design. The Smooth Evolutionary Structural Optimization (SESO) method was proposed by [7] and is based on the ESO method. It uses a smoothing technique to improve the convergence rate of the optimization algorithm and to reduce the number of iterations required to reach an optimal design.

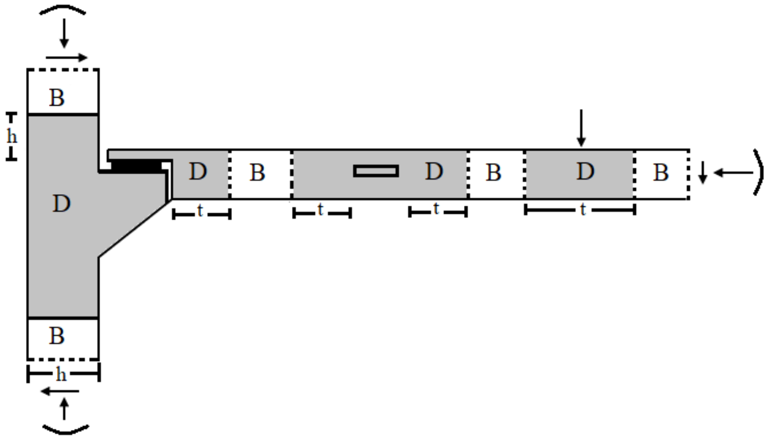

Figure 1. Examples of D and B regions

STMs must be arranged in such a way that there is coincidence between the center of gravity of each element and the lines of action of the external forces acting on each node. In addition, these models must meet some criteria, including the balance of internal stresses, considering the load, which must not exceed the limits in relation to the actual strength of the structure, see [8] and [1].

The traditional RBTO approaches are called double-loop (or nest-loop) approaches, where the reliability constraints are implemented to make the optimization problems measurable and solvable using the first-order reliability method (FORM). One of the best-known approaches based on the FORM method is the Reliability Index Approach (RIA). However, the double-loop RBTO approach is computationally expensive and lacks robustness when dealing with a large number of random variables [9]. To overcome this, single-loop approaches have been developed, where the Karush-Kuhn-Tucker (KKT) optimization conditions are used to avoid the loop. Authors such as [10], [11], [12], and [13] have used the double-loop formulation, while [14] developed a proprietary single-loop model called the RBTO-hybrid method that solves the reliability analysis and design optimization problems simultaneously to find the global solution, i.e., the most probable point (MPP), with low computational cost. Other approaches that use the single-loop formulation include [15], which uses a decoupling approach, or [16] and [17].

Although RBTO is a rapidly expanding field of research, the relationship between TO and probability constraint is still quite challenging, due to the difficult task of assessing the probability of failure in a direct estimate. There are still few works that integrate the concept of reliability to 3D structures, we can mention [18] who address RBTO combining Sequential Optimization and Reliability Assessment (SORA) with an external optimization software. In [19], the RBTO with the BESO optimization procedure is proposed and [20] uses the Segmental Multi-Point Linearization (SML) method for a more accurate estimate of the failure probability gradient. The methodology of using RBTO for 3D structures with SESO method and the sequential element rejection and admission (SERA) were presented by [21]. In this sense, the present paper becomes a contribution to the field of structural reliability applied to three-dimensional structures. In addition, it brings a simple methodology for automated generation of models of connecting rods and tie rods obtained through the partial derivatives of the von Mises stress field, allowing the knowledge of the regions of preponderance of traction (blue) and compression (green).

2 Topology optimization of STMs

2.1 Statement of the TO problem in stress

The objective of the strut-and-tie analysis via topology optimization strategy is to find a reinforcement layout within the design domain that minimizes the maximum von Mises stress of the structure for some given loading and boundary conditions for linear elastic problems. The following formulation is typically used:

$$\begin{aligned} &minimize \quad V=\sum_{e=1}^{ne} x_e V_e \\ &subject\ to\!: \ KU=F \\ &\qquad\qquad\quad \sigma_e^{vm}-\sigma^{*}\leq 0 \\ &\qquad\qquad\quad x_e=0\ or\ x_e=1 \end{aligned} \tag{1}$$

where the von Mises stress σ_e^{vm} on each element is calculated using equation 2.

$$\sigma_e^{vm}=\left[\sigma_x^2+\sigma_y^2+\sigma_z^2-\sigma_x\sigma_y-\sigma_x\sigma_z-\sigma_y\sigma_z+3\tau_{xy}^2+3\tau_{xz}^2+3\tau_{yz}^2\right]^{1/2} \tag{2}$$

where V is the volume of the whole structure, V_e is the volume of the *e-th* element, K is the stiffness matrix of the structure, U is the displacements vector , F is the force vector, *ne* is the total number of finite elements of the structure, σ_e^{vm} is the von Mises stress of element e, σ^{*} is an admissible stress, $x_e=0$ denotes empty material and $x_e=1$ denotes solid material. This formulation shows that the optimization procedure aims to minimize the amount of elements and therefore minimize the volume of the structure. This structure is subject to the equilibrium equations as well as a stress constraint for each element that must be less than or equal to the permissible stress.

2.2 Sensitivity analysis for automated generation of STMs models

Taking the local calculation of the derivative of the von Mises stress of the element with respect to the components of the stress vector described respectively as:

$$\begin{aligned} \frac{\partial(\sigma_e^{vm})}{\partial\sigma_x}&=\frac{1}{2\sigma_e^{vm}}\left(2\sigma_x-\sigma_y-\sigma_z\right) \\ \frac{\partial(\sigma_e^{vm})}{\partial\sigma_y}&=\frac{1}{2\sigma_e^{vm}}\left(2\sigma_y-\sigma_x-\sigma_z\right) \\ \frac{\partial(\sigma_e^{vm})}{\partial\sigma_z}&=\frac{1}{2\sigma_e^{vm}}\left(2\sigma_z-\sigma_x-\sigma_y\right) \end{aligned} \tag{3}$$

Considering equation 3 and making $\frac{\partial(\sigma_e^{vm}(x))}{\partial\sigma_x}>0$ then the elements are preponderantly tensioned (blue color - ties) while $\frac{\partial(\sigma_e^{vm}(x))}{\partial\sigma_z}<0$ are preponderantly compressed (green color - strut).

3 Short review of reliability-based topology optimization

RBTO-based design measures the uncertainty of the structure by probability of failure or reliability index. Thus, it is able to improve the performance of the structure by reducing its failure probability. This paper aims at developing an efficient RBTO algorithm applied to automated generation of STMs. It considers the random variables: Force, Geometry, Volume and Compliance. It uses the hybrid reliability analysis model, proposed by [6] for 2D structures and it has been expanded in the present paper for 3D structures.

3.1 Mathematical Model of RBTO

For the mathematical model of RBTO it is sufficient to transform the stress constraint in eq. (1) as follows:

$$\begin{aligned}
&\textit{minimize:} && V(x_i, \boldsymbol{X}_j, \boldsymbol{u}) = \textstyle\sum_{i=1}^{NE} x_i V_i(x_i, \boldsymbol{X}_j, \boldsymbol{u}) \\
&\textit{subject to:} && P_S(\mathbf{X}) = \mathrm{P}\left[G\left(x_i, \mathbf{X}_j\right) \leq 0\right] \geq P_t \\
& && P_f = \mathrm{P}\left[G\left(x_i, \mathbf{X}_j\right) \leq 0\right] = \int \dots \int_{G(x_i, X) \leq 0} f_X(\mathbf{X}) dx \\
& && \mathrm{K}(x_i, \boldsymbol{X}_j, \boldsymbol{u}) \mathrm{U}(x_i, \boldsymbol{X}_j, u) = \mathrm{F}(\boldsymbol{X}_j, \boldsymbol{u}) \\
& && \beta(\boldsymbol{u}) = \beta_t \\
& && x_i = 1 \quad or\; x_i = 10^{-9} \quad \text{with} \quad i = 1, \dots, NE \;\; and \;\; j = 1, \dots, m
\end{aligned} \tag{4}$$

with x_i being the finite element, $\mathbf{X}_j$ is the *j-th* random variable, V is the volume of the total structure, P_S is the probability of success, P_t is the target probability of success, G is the limit state function, *NE* is the number of variables and *m* the number of uncertain variables. To control the topologies obtained by the RBTO model the reliability index $\beta(\boldsymbol{u})$, see [6], is introduced with a normalized vector $\boldsymbol{u}$**.**

$$G\left(x_i, \mathrm{X}_j\right) = \sigma^* - \sigma_e^{vm}(x_i, \mathrm{X}_j) \tag{5}$$

σ^* indicates the allowable stress for the material and $\sigma_e^{vm}(x_i, \mathbf{X}_j)$ indicates the von Mises stress of the element Thus, if $G > 0$, the structure is reliable, if $G < 0$ the structure failure and if $G = 0$ the structure is in the limit state.

4 Numerical Examples

4.1 Example 1 – Simply supported deep beam with two openings

A simply supported deep beam with two openings is illustrated in figure 2. Two concentrated loads of magnitude $F = 140\ kN$, dimensions 120x60x10 (cm) totaling 67,500 hexahedral finite elements, Poisson's ratio of $\nu = 0.30$, Young's modulus of concrete $E = 30{,}088\ GPa$ with $lx = 120\ cm,\ ly = 60\ cm,\ lz =$ 10cm, $a = 5cm, d = 15\ cm, c = 10cm$ A rejection ratio $RR = 1\%$ and an evolutionary ratio $ER = 1\%$ are considered. The optimal settings is presented in figure 3a and the automatically generated strut-and-tie model is presented in figure 3b. It can be observed that the D-regions of the design have direct influence on the load transfer mechanism as the natural path of the load is redirected around the opening. This is confirmed by the STMs, see fig. 3b, which shows that the loads are transmitted to the supports by the struts (green region: preponderance of compression) around the openings. Note also that the two inclined ties (blue region: preponderance of tensile stresses) around the openings connect the upper and lower struts around the opening. Highlighted that, the two openings have 225 cm² in their cross section.

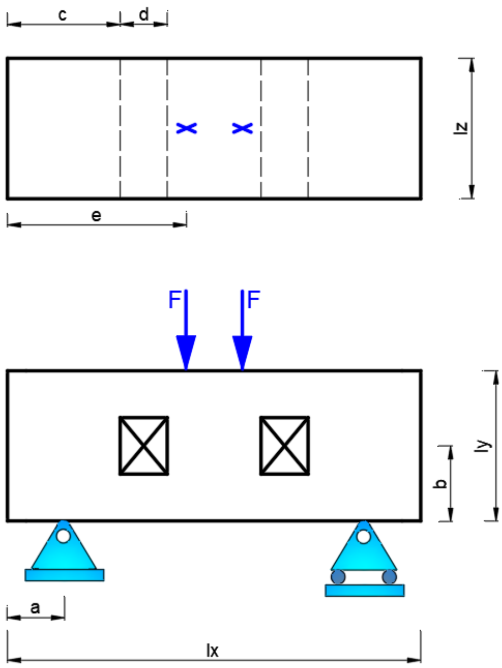

Figure 2 – Designs domain of the deep beam with two openings

The initial design parameters are given in Table 1, where nelx (length), nely (height), and nelz (width) represent the structure's geometry, F, which represents the external concentrate load, and they are considered random variables with normal distribution, while the material properties have constant distribution. The standard deviation values are obtained as indicated in the [22]

Table 1. Coefficients in constitutive relations

Distribution parameter	Distribution type	Mean (μ)	Standard deviation (σ)
nelx(cm)	Normal	120	0.1
nely(cm)	Normal	60	0.1
nelz(cm)	Normal	10	0.1
E(GPa)	Normal	30.088	0.1
ν	Constant	0.30	0
F (kN)	Normal	140	0.1
Volume (cm^3)	Normal	0.40	0.1
Compliance (kN.cm)	Normal	4.978e3	0.1
f_{ck}(MPa)	Constant	35	0
f_{yd}(MPa)	Constant	435	0

Note also that simply supported deep beam with two openings, when the $\beta_t = 3.0$, that is, failure probability $P_f = 0.001358$, it takes only 4 iterations to reach convergence in the reliability calculation. Thus, the proposed approach only needs to evaluate the Finite Element Analysis (FEA) 51 times, while the traditional RBTO method needs to process FEA 204 times. Therefore, the proposed approach is shown to have higher efficiency. Furthermore, the final volume in the RBTO analysis was reduced by 9.5% reaching the volume of 0.362

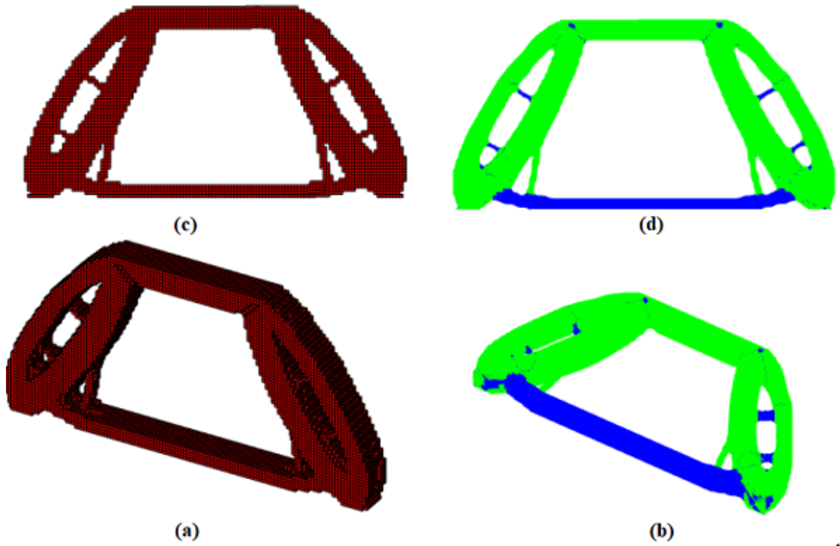

Figure 3 – Deep beam with two openings: (a) Topology optimal RBTO-SESO (c) RBTO-SESO front (b) RBTO - Strut-and-tie model with present formulation and (d) Strut-and-tie front

5 Conclusions

This paper introduced a groundbreaking strategy for reliability-based topology optimization (RBTO) merged with the Smoothing-ESO (SESO) technique for the automated creation of optimal strut-and-tie models in 3D structures. The methodology was effectively applied to a deep beam with two openings, considering random variables like geometry, volume, strength, and compliance. The reliability analysis was conducted using the Reliability Index Approach (RIA) through the First-Order Reliability Method (FORM). The automatic generation of 3D strut-and-tie models was accomplished by using the derivatives of the von Mises stress fields, which supplied data on the force paths where compression and tension predominantly occurs. This information was utilized to direct the reinforcement placement, resulting in optimal models for the specified structural issues. The suggested RBTO-SESO approach demonstrated efficiency and robustness, offering a substantial reduction in computing time compared to traditional RBTO methods. This research adds to the field of structural reliability applied to three-dimensional structures and presents a novel methodology for the automated creation of strut-and-tie models. Future research in this domain could investigate the application of the proposed strategy to an expanded array of structural problems and incorporate other advanced optimization algorithms and reliability techniques for an even more effective and reliable design process.

Acknowledgment

The authors acknowledge the Sao Paulo State Research Foundation (FAPESP) under Grant Number 2016/02327-5 for their financial support and CNPq (National Council of Scientific and Technological Development) under Grant Numbers 305093/2018-5.

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